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FLOW OF FREEZING WATER AND AQUEOUS SALT SOLUTIONS THROUGH TUBES

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The problem of internal freezing of pipes is formulated and results of numerical studies presented.

Transport of freezing liquids, including water, various suspensions, and aqueous solutions of salts under conditions of reduced ambient temperature may be accompanied by internal icing of lines [1], which leads to an increase in the pressure head required to pump the liquid, or complete closing of the "live" line section and impossibility of further use.

We will consider the process of ice freezing on the inner wall of a tube when aqueous solutions of certain salts, for example, sodium chloride, flow therein. We will assume that the salt concentration in the water is less than eutectic (for sodium chloride the eutectic concentration in water is approximately 30% by weight [2]).

In freezing of a subeutectic composition only the solvent goes through a phase transition. The concentration of the dissolved material will then increase near the phase transition boundary. Due to the difference between concentrations near the phase transition boundary and the remaining part of the solution which thus develops the dissolved substance will be removed from the phase transition zone by diffusion.

Thus, the salt concentration at the phase transition boundary and the related freezing temperature will be variables, and the process of salt water freezing during flow in a tube must be described by combined heat and mass transport equations. The formulation of the problem will be analogous to that of [3].

For flow of an incompressible liquid in a tube of varying section the continuity equation can be written in the following form:

$$\frac{\partial S_w}{\partial t}+\frac{\partial (VS_w)}{\partial x}=\frac{\rho_\text{т}}{\rho_{\ell}}\,\frac{\partial S_w}{\partial t}. \tag{1}$$

In the case under consideration the equation of motion has the form

$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}=-\frac{1}{\rho_{\ell}}\frac{\partial P}{\partial x}-\frac{\sqrt{\pi}\,\xi V^2}{4\sqrt{S_w}}-g\sin\gamma-\frac{\rho_\text{т}}{\rho_{\ell}}\,\frac{V}{S_w}\,\frac{\partial S_w}{\partial t}. \tag{2}$$

The energy equation can be written as

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$$\frac{\partial T}{\partial t}+V\frac{\partial T}{\partial x}=\frac{\lambda_\ell}{C_\ell \rho_\ell S_w}\frac{\partial}{\partial x}\left(S_w\frac{\partial T}{\partial x}\right)+\frac{2\alpha\sqrt{\pi}(T_w-T)}{C_\ell\rho_\ell\sqrt{S_w}}+\frac{\sqrt{\pi}\,\xi V^3}{4C_\ell\sqrt{S_w}}. \tag{3}$$

The continuity equation for the salt component takes on the form

$$\frac{\partial}{\partial t}(S_w\Theta)+\frac{\partial}{\partial x}(VS_w\Theta)=0. \tag{4}$$

The solution freezing point T_w depends [2] on the concentration at the phase transition boundary

$$T_w=T_1(1-k\Theta_w). \tag{5}$$

On the phase boundary we write the thermal balance

$$\frac{dS_w}{dt}=\frac{\dfrac{4\pi\lambda_т(T_w-T_н)}{\rho_т L}}{\ln\dfrac{S_w}{S_0}-\dfrac{2\lambda_т}{\lambda_{gr}G}-\dfrac{2\lambda_т}{\lambda_{u.s.c}}\sqrt{\dfrac{S_0}{S_w}}\ln\left(1+\dfrac{\delta_{u.s.c.}}{R_0}\right)}-\frac{2\alpha\sqrt{\pi S_w}(T_w-T)}{\rho_т L} \tag{6}$$

and salt mass

$$\frac{dS_w}{dt}=-2\beta\frac{\rho_\ell}{\rho_т}\sqrt{\pi S_w}\left(1-\frac{\Theta}{\Theta_w}\right). \tag{7}$$

With consideration of Eq. (5), we obtain from Eqs. (6) and (7) an expression for the unknown salt concentration on the phase transition boundary

$$\Theta_w=\frac{b-\sqrt{b^2-4a\Theta}}{2a}, \tag{8}$$

where

$$a=\frac{kT_1}{\beta\rho_\ell L}\left[\frac{\dfrac{2\sqrt{\pi}\lambda_т}{\sqrt{S_w}}}{\ln\dfrac{S_w}{S_0}-\dfrac{2\lambda_т}{\lambda_{gr}G}-\dfrac{2\lambda_т}{\lambda_{u.s.c}}\sqrt{\dfrac{S_0}{S_w}}\ln\left(1+\dfrac{\delta_{u.s.c}}{R_0}\right)}-\alpha\right];$$

$$b=\frac{\dfrac{2\sqrt{\pi}\lambda_т(T_1-T_н)}{\beta\rho_\ell L\sqrt{S_w}}}{\ln\dfrac{S_w}{S_0}-\dfrac{2\lambda_т}{\lambda_{gr}G}-\dfrac{2\lambda_т}{\lambda_{u.s.c}}\sqrt{\dfrac{S_0}{S_w}}\ln\left(1+\dfrac{\delta_{u.s.c.}}{R_0}\right)}-\frac{\alpha(T_1-T)}{\beta\rho_\ell L}+1.$$

We drop the + sign before the root, since Θ_w cannot be negative.

Substituting Eq. (5) in Eq. (6), we obtain

$$\frac{dS_w}{dt}=\frac{2\sqrt{\pi S_w}\beta\rho_\ell}{\rho_т}[(b-1)-a\Theta_w].$$

Here the first term indicates the freezing rate in the case of a flow of pure water, while the second is the correction for salinity.

Since the salinity correction is opposite in sign to the first term, the freezing rate for flow of salt water will be lower than for fresh water flow.

We will consider a quasisteady state flow regime with a specified constant volume flow rate.

In that case Eqs. (1)-(4) take on the form

$$VS_w = Q = \text{const},$$
$$\frac{dP}{dx} = -\rho_{\ell} g \sin\gamma - \frac{\sqrt{\pi}\,\xi\rho_{\ell} Q^2}{4S_w^{2,5}},$$
$$\frac{dT}{dx} = \frac{2\alpha\sqrt{\pi S_w}\,[T_1(1-k\Theta_w)-T]}{C_{\ell}\rho_{\ell} Q} + \frac{\sqrt{\pi}\,\xi Q^2}{4C_{\ell} S_w^{2,5}},\tag{9}$$
$$\frac{d\Theta}{dx} = \frac{2\beta\sqrt{\pi S_w}}{Q}\left(1-\frac{\Theta}{\Theta_w}\right).$$

In deriving these expressions in the equation of motion we have neglected the velocity head, and in the energy equation we have ignored the liquid thermal conductivity along the tube axis.

In the dimensionless variables $\bar{x} = \frac{x}{l}$, $\bar{P} = \frac{P}{P_0}$, $\bar{T} = \frac{T}{T_0}$, $\bar{S}_w = \frac{S_w}{S_0}$, $\bar{\Theta} = \frac{\Theta}{\Theta_0}$, $\bar{\Theta}_w = \frac{\Theta_w}{\Theta_0}$, $\tau = \frac{4\pi\lambda_T T_0}{\rho_T L S_0} t$ the system of equations describing the process of freezing of salt water in tubes can be written in the following manner (we now omit the bar above the dimensionless variables:

$$\frac{dP}{dx} = -\frac{\rho_{\ell} g l \sin\gamma}{P_0} - \frac{\xi\rho_{\ell} l Q^2}{4\pi^2 R_0^5 P_0 S_w^{2,5}},$$
$$\frac{dT}{dx} = \frac{2\pi\alpha l R_0}{C_{\ell}\rho_{\ell} Q}\left[\frac{T_1}{T_0}(1-k\Theta_0\Theta_w) - T\right]\sqrt{S_w} + \frac{\xi l Q^2}{4\pi^2 C_{\ell} R_0^5 T_0 S_w^{2,5}},$$
$$\frac{d\Theta}{dx} = \frac{2\beta l\pi R_0}{Q}\,\Theta\sqrt{S_w}\left(1-\frac{\Theta}{\Theta_w}\right),$$
$$\frac{dS_w}{d\tau} = -\sqrt{S_w}\left(1-\frac{\Theta}{\Theta_w}\right),\quad \Theta_w = \frac{b-\sqrt{b^2-4a\Theta_0\Theta}}{2a\Theta_0},$$
$$a = \frac{2\lambda_T k T_1}{\beta\rho_{\ell} L R_0}\left[\frac{1}{\sqrt{S_w}\,(\ln S_w - 2\lambda_1 - 2\lambda_2/\sqrt{S_w})} - \frac{\text{Bi}}{2}\right],\tag{10}$$
$$b = \frac{2\lambda_T T_1}{\beta\rho_{\ell} L R_0}\left[\frac{1-\dfrac{T_{\text{H}}}{T_1}}{\sqrt{S_w}\left(\ln S_w - 2\lambda_1 - \dfrac{2\lambda_2}{\sqrt{S_w}}\right)} - \frac{\text{Bi}}{2}\left(1-\frac{T_0 T}{T_1}\right)\right] + 1,$$
$$\lambda_1 = \frac{\lambda_T}{\lambda_{\text{gr}} G},\quad \lambda_2 = \frac{\lambda_T}{\lambda_{\text{u.s.c.}}}\ln\left(1+\frac{\delta_{\text{u.s.c.}}}{R_0}\right),\quad \text{Bi} = \frac{\alpha R_0}{\lambda_T}.$$

The thermal resistance of the disturbed ground is defined in the following manner [1]:

$$G = \max\left\{\frac{1}{\ln\left(\dfrac{H}{R_0} + \sqrt{\left(\dfrac{H}{R_0}\right)^2 - 1}\right)};\ \frac{R-1}{R\ln R - R + 1}\right\},$$

and the radius of the thermal disturbance is found by solution of the differential equation

$$\frac{dR}{d\tau} = \frac{\varkappa_{\text{b}}\rho_T}{2\beta\rho_{\ell} R_0}\,\frac{\ln R + \dfrac{1}{R} - 1}{\dfrac{R\ln R}{6}\left(1+\dfrac{1}{R}+\dfrac{1}{R^2}\right) - \dfrac{R}{4} + \dfrac{1}{4R}}.\tag{11}$$

The heat liberation coefficient from the liquid to the wall can be calculated from the expression of [4]

$$\mathrm{Nu} = 0{,}021\,\mathrm{Re}^{0,8}\,\mathrm{Pr}^{0,43}$$

or from the equation presented in [5]

$$\alpha = 0{,}031\rho_\ell C_\ell \sqrt{\xi}\,\mathrm{Pr}^{-0,75}.$$

The mass liberation coefficient can be found from the similarity of the heat and mass exchange processes [6]:

$$\beta = \frac{\alpha}{C_\ell \rho_\ell}.$$

We define the hydraulic resistance coefficient from the expression

$$\xi = (1{,}8\lg\mathrm{Re} - 1{,}5)^{-2}.$$

The algorithm for computation consists of the following steps:

1) for values of the soil disturbance radius R and "live" section area of the tube S_w fixed in time, with consideration of the initial conditions

$$P(0) = 1,\ T(0) = 1,\ \Theta(0) = 1$$

we calculate the distributions of pressure, temperature, and concentration over the length of the line;

2) for a given distribution P(x), T(x), Θ(x), we make a step in time and using the initial conditions R(0) = 1, $S_w(0) = 1$, we find the new value of the ground thermal disturbance radius R and "live" section area $S_w(x)$;

3) for the following moment in time we again find P(x), T(x), Θ(x), then redetermine R and $S_w(x)$ and so on.

Ice formation on the tube walls is possible only under certain conditions. According to Eqs. (7) and (8) we can write the general condition in the following manner:

$$\frac{b - \sqrt{b^2 - 4a\Theta}}{2a} > \Theta.$$

Then, using the expression for a and b, we obtain the condition for ice formation on the inner surface of the tube

$$\tilde{T} = \frac{T - T_\text{н}}{T_1 - T_\text{н}} < \left(1 - \frac{\dfrac{2}{\mathrm{Bi}\sqrt{S_w}}}{\ln S_w - 2\lambda_1 - \dfrac{2\lambda_2}{\sqrt{S_w}}}\right)\left(1 - \frac{kT_1\Theta_0\Theta}{T_1 - T_\text{н}}\right). \tag{12}$$

Using Shukhov's expression for temperature distribution along the length of the tube

$$T(x) = \frac{T_\text{н}}{T_0} + \left(1 - \frac{T_\text{н}}{T_0}\right)\exp\left(-\frac{2\alpha^*\pi R_0 l x}{C_\ell \rho_\ell Q}\right)$$

and substituting in Eq. (12), we find the conditions for which ice formation begins on the inner surface of the tube

$$\frac{T_0 - T_\text{н}}{T_1 - T_\text{н}}\exp\left(-\frac{2\alpha^*\pi R_0 l x}{C_\ell \rho_\ell Q}\right) < \left(1 - \frac{kT_1\Theta_0\Theta}{T_1 - T_\text{н}}\right)\left[1 + \frac{1}{\mathrm{Bi}(\lambda_1 + \lambda_2)}\right]. \tag{13}$$

Condition (13) permits solution of a number of problems of practical importance: 1) for given input and external temperatures, thickness and composition of thermal insulation, method of line installation – underground or above-ground ($G \to \infty$) one can determine the coordinate x_i of the line where the ice layer begins to form; 2) for given line parameters the minimum insulation thickness at which ice formation is eliminated over the entire length of the tube (x = 1) can be determined; 3) for given temperatures, line and insulation parameters, the minimum depth to which a line must be buried to avoid ice formation can be found. These problems can be solved for flow of either salt or fresh (Θ = 0) water.

It is evident from inequality (12) that for commencement of ice formation on the tube walls for salt water flow it is necessary to create a more severe supercooling than for a pure water flow. Thus, for an above-ground line and salt water flow the external temperature must be $\Delta T_H = kT_1\Theta_0\Theta$ degrees less than for fresh water flow with other conditions equal.

At certain combinations of parameter values formation of an ice layer of constant thickness is possible. The steady state position of the "live" section area can be determined by solving the equation

$$\sqrt{S_w}(\ln S_w - 2\lambda_1) - 2\lambda_2 = \frac{2}{\mathrm{Bi}}\,\frac{1 - k\Theta_0\Theta - T_{\text{н}}/T_1}{1 - k\Theta_0\Theta - T_0 T/T_1},$$

obtained from Eq. (8) for the condition $\Theta_w = \theta$.

For flow of fresh water in an above-ground line the steady state position of the "live" section can be found from the condition

$$\sqrt{S_w}\ln S_w = \frac{2}{\mathrm{Bi}}\,\frac{1 - T_{\text{н}}/T_1}{1 - T_0 T/T_1}.$$

A layer of ice which has formed can be removed by increasing the input temperature of the flow. In that event both the temperature of the flow core and the temperature on the phase transition boundary increase, causing decomposition of the ice layer.

By dropping Eqs. (4), (7), (8) and considering that for pure water $T_w = T_1$, we obtain the problem of flow of a freezing liquid formulated in [1]. As an example, we will calculate flow of water in a line with the following parameters: line length 150 km, diameter 0.418 m, installation depth 1.5 m, water flow rate 0.206 m^3/sec, soil thermal conductivity and diffusivity 1.7 W/(m·deg) and $0.77 \cdot 10^{-6}$ m^2/sec.

The results of the fresh water flow calculation are shown in Fig. 1. The initial water temperature and external ground temperature have a significant effect on the ice formation process. At low temperatures the ice layer begins to grow along the entire line length (Fig. 1b), while at higher temperatures the ice begins to form only at some initial section (Fig. 1a). With the passage of time this section shifts toward the tube mouth as the thickness of the ice layer increases. After enough time passes the line may completely clear itself.

The ice formation process occurs somewhat differently in the presence of salts dissolved in the water. Calculations were performed for the following parameter values: line length 4 km, diameter 0.25 m, salt solution flow rate 0.3 m^3/sec, line installation depth 2 m, soil thermal conductivity coefficient 2.1 W/(m·deg), input temperature 273 K, external temperature 263 K. The presence of salt first decreases (Fig. 2) the ice formation rate and may even completely prevent the process. Thus, flow of a 5% salt solution at the same temperatures as presented in Fig. 1a is accompanied by only short-term insignificant ice formation, and even at $\tau = 0.2$ the tube clears itself completely.

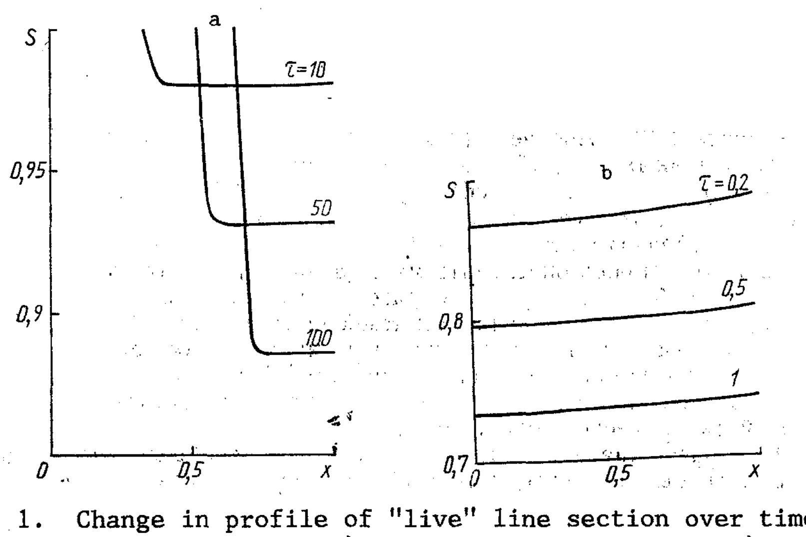

Fig. 1. Change in profile of "live" line section over time for flow of fresh water: a) T_0 = 277 K, T_H = 267 K; b) 273 and 263 K.

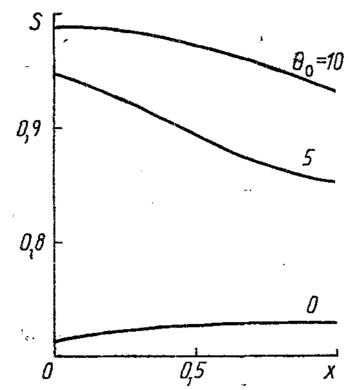

Fig. 2

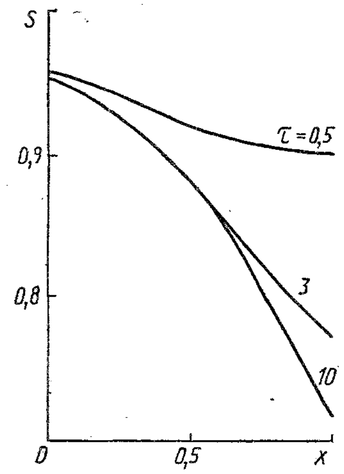

Fig. 3

Fig. 2. Profiles of "live" line section at time $\tau = 1$ for various initial salt concentrations (mass %).

Fig. 3. Change in profile of "live" line section with time for initial concentration $\Theta_0 = 5$.

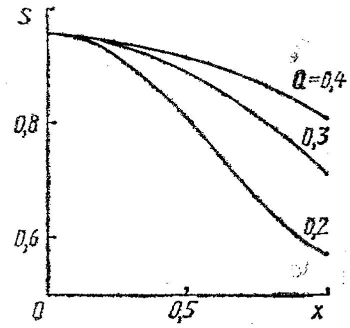

Fig. 4. Profiles of "live" line section at time $\tau = 10$ for various liquid flow rates.

Second, because the phase transition temperature changes along tube length, the profile of the "live" section is steeper than for fresh water flow. In this case the ice thickness increases toward the tube mouth (Fig. 3), while for fresh water the thickness decreases (see Fig. 1b).

Third, a steady state ice layer is established in the tube quite quickly, and cannot be removed without changing the flow parameters. As was noted above, removal of an established ice layer is possible by increasing the flow input temperature or the external soil temperature, or the salt concentration in the flow, which is equivalent to raising the external temperature by $\Delta T_H = kT_1\Delta\Theta$ degrees. Increase in liquid flow rate also leads to decreased danger of ice formation (Fig. 4). Thus, increase in liquid flow from 0.2 to 0.5 m^3/sec completely eliminates ice formation. However in this case the hydraulic resistance of the tube increases by a factor of more than two.

Calculations also showed that for other conditions equal the process of ice formation depends significantly on the method of installing the line. For example, underground installation at a depth of 2 m leads to self-clearing over a time $\tau = 0.2$, while for on-ground installation the ice formation process continues. To prevent ice formation it is necessary to bury or thermally insulate the line.

Thus the problem of flow in tubes of freezing aqueous salt solutions has been formulated. Numerical results have been presented for a quasisteady state regime. The different character of the process for flows of fresh and salt water has been shown. An expression of practical importance has been obtained, allowing determination of flow conditions for both fresh and salt water at which internal icing of the line will be excluded.

NOTATION

P, T, V, S_w, pressure, temperature, velocity, and area of "live" flow section; Θ, salt concentration in flow; ρ_T, ρ_ℓ, density of ice and solution; C_ℓ, specific heat of the solution; λ_T, λ_ℓ, λ_{gr}, thermal conductivity coefficients of ice, solution, and ground; α, heat liberation coefficient from liquid to the tube wall; α*, heat liberation coefficient from liquid to ground; ξ, hydraulic resistance coefficient; T_H, external temperature; L, heat of phase transition; R_0, S_0, radius and area of tube cross section; G, thermal resistance of disturbed ground; δ_{in}, λ_{in} thickness and thermal conductivity coefficient of insulation.

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COMPREHENSIVE INVESTIGATION OF STARTUP REGIMES FOR A FROZEN HEAT PIPE

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The authors describe results of an experimental investigation of startup of an ammonia heat pipe made of aluminum with longitudinal channels and radiative heat rejection. They determine the influence of the startup dynamics of the location of the frozen heat transfer agent and the slope of the heat pipe to the horizontal. They propose a simplified method of determining the startup characteristics when the main mass of heat transfer agent is frozen in the heater zone.

The startup of low-temperature heat pipes is characterized by a number of special features which give rise to certain problems. These include the possible startup of a heat transfer agent from the frozen state with a given means of heat supply and removal, and the need to account for possible operation of the heat pipe at the hydrodynamic power limit. Startup of a heat transfer agent from the frozen state is a sequence of complex physical processes involving phase transitions of substances and unsteady effects.

The dynamic characteristics of low-temperature heat pipes have been investigated in a number of papers [1-3], where, in the main, startup from a state with a liquid heat transfer agent was investigated. One should note the theoretical paper [4], which formulated the problem of a frozen heat pipe with a homogeneous wick, and also papers [5, 6] where theoretical and experimental data for startup of a water heat pipe were examined.

The aim of the present paper is a comprehensive investigation of startup conditions of an ammonia heat pipe when heat is supplied to the evaporator by convection to the liquid and heat is removed by radiation.

The heat pipe had 45 longitudinal open grooves of depth $0.82 \cdot 10^{-3}$ m and width $0.5 \cdot 10^{-3}$ m. The material of the body was aluminum. The geometric dimensions of the pipe were: total length 1.72 m, length of the heater zone 0.2 m, length of the cooler zone 1.48 m,

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